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II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let  $ACB$  be the triangle; choose  $BC=a$  for the axis of abscisses, and  $CA$  for that of ordinates. Any circumscribed ellipse is of the form  $y^2 + Axy + Bx^2 + Cy + Dx = 0$ ; and since  $(a, 0)$  and  $(0, b)$  are points of the ellipse, we have  $C = -b$ ,  $D = -Ba$ , and the above equation reduces to  $y^2 + Axy + Bx^2 - by - Bax = 0$ .

Transforming it to the center of the ellipse as origin, it reduces to

$$y^2 + Axy + Bx^2 - \frac{Bb^2 - ABab + B^2a}{4B - A^2} = 0.$$

The area is  $= \pi \sin C$ .  $\frac{Bb^2 - ABab + B^2a}{4B - A^2} = m$ . Developing  $\frac{\partial m}{\partial A} = 0$ , and  $\frac{\partial m}{\partial B} = 0$ , we find  $A = \frac{b}{a}$  and  $B = \frac{b^2}{a^2}$ , and thus find the ellipse of minimum area to be,  $a^2y^2 + abxy + b^2x^2 - a^2by - ab^2x = 0$ . The center is the point  $(a/3, b/3)$ . The maximum ellipse about the triangle is concentric, and its equation is

$$a^2y^2 + abxy + b^2x^2 - a^2by - ab^2x + \frac{a^2b^2}{4} = 0.$$

Also solved by C. N. Schmall, and V. M. Spunar.

280. Proposed by C. N. SCHMALL, 89 Columbia Street, New York.

Find the envelope of the system of spheres  

$$\left. \begin{aligned} (x-a)^2 + (y-b)^2 + z^2 &= r^2 \\ a^2 + b^2 &= c^2 \end{aligned} \right\}.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; V. M. SPUNAR, Pittsburg, Pa., and the PROPOSER.

Differentiating  $(x-a)^2 + (y-b)^2 + z^2 = r^2$  and  $a^2 + b^2 = c^2$  with reference to  $a$  as the independent variable, we have  $(x-a) + (y-b) \frac{\partial b}{\partial a} = 0$ , and  $a + b \frac{\partial b}{\partial a} = 0$ ; from the second equation we get  $\frac{\partial b}{\partial a} = -\frac{a}{b}$ , and substituting in the first we get  $(x-a) - \frac{a}{b}(y-b) = 0$ , whence  $b = \frac{ay}{x}$  and combining this with  $a^2 + b^2 = c^2$ , we get  $a^2 = \frac{c^2x^2}{x^2 + y^2}$ ,  $b^2 = \frac{c^2y^2}{x^2 + y^2}$ , and substituting this in the first given equation, we have, after some easy reductions:

$$x^2 + y^2 + z^2 - 2c\sqrt{(x^2 + y^2)} = r^2 - c^2.$$

This is, as can easily be seen, the equation of the surface of a ring, the central line of which is a circumference whose radius is  $=c$ , and the perpendicular section of a circle whose radius is  $=\sqrt{(2c^2 - r^2)}$ .